Chapter 4

Bipolar Junction Transistors. Problem Solutions

4.1 Problem 4.37

It is required to design the circuit in Figure (4.1) so that a current of 1 mA is established in the emitter and a voltage of +5 V appears at the collector. The transistor type used has a nominal β of 100. However, the β value can be as low as 50 and as high as 150. Your design should ensure that the specified emitter current is obtained when $\beta = 100$ and that at the extreme values of β the emitter current does not change by more than 10% of its nominal value. Also, design for as large value for R_B as possible. Give the values of R_B, R_E, and R_C to the nearest kilo-ohm. What is the expected range of collector current and collector voltage corresponding to the full range of β values?

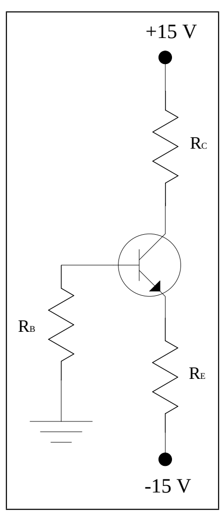

Figure 4.1:

Solution

Nominal $\beta = 100$, so nominal $\alpha = \beta/(1+\beta) = 0.99$, nominal $I_E = 1$ mA, nominal $I_C = \alpha I_E$ $= 0.99$ mA, nominal $V_C = +5$ V. R_C can then be calculated as:

$$\begin{aligned} R_C &= \frac{V_{CC} - V_C}{I_C} \\ &= \frac{15 - 5}{0.99} \\ &= 10.1\ k\Omega \\ &= 10\ k\Omega \end{aligned}$$

Applying Kirchhoff's voltage rule on the base-emitter loop we get:

$$I_E R_E + I_B R_B = V_{EE} - V_{BE}$$

Using $I_B = I_E/(\beta + 1)$, we then get:

$$\begin{aligned} I_E &= \frac{V_{EE} - V_{BE}}{R_E + \frac{R_B}{1+\beta}} && (4.1) \\ &= \frac{15 - 0.7}{R_E + \frac{R_B}{101}} \\ 1 &= \frac{14.3}{R_E + \frac{R_B}{101}} \\ R_E + \frac{R_B}{101} &= 14.3 && (4.2) \end{aligned}$$

The collector current depends only on V_{BE}, while I_B and I_E depends also on β. Note that for the same collector current, changing β from 100 to 50 changes the base current by a factor of 2, while changing it from 100 to 150, changes the base current by a factor 2/3. This means that reducing β will have more effect on the emitter current then increasing it. So we design the circuit to limit the maximum change in the emitter current at $\beta = 50$. Since decreasing β decreases the emitter current we then use the lower limit of I_E of 0.9 mA and $\beta = 50$ in Equation (4.1):

$$\begin{aligned} 0.9 &= \frac{14.3}{R_E + \frac{R_B}{51}} \\ R_E + \frac{R_B}{51} &= 15.89 && (4.3) \end{aligned}$$

Using Equation (4.2) and Equation (4.3) we get:

$$\begin{aligned} R_B &= 164\ k\Omega \\ R_E &= 13\ k\Omega \end{aligned}$$

to find the range of I_C and V_C for the full range of β values we use:

$$\begin{aligned} I_C &= \alpha I_E \\ &= \frac{\beta}{1+\beta} \times \frac{V_{EE} - V_{BE}}{R_E + \frac{R_B}{1+\beta}} \qquad (4.4) \\ V_C &= V_{CC} - I_C R_C \qquad (4.5) \end{aligned}$$

Using Equation (4.4) and Equation (4.5) we get for $\beta = 50$:

$$\begin{aligned} I_C &= \frac{50}{1+50} \times \frac{15 - 0.7}{13 + \frac{164}{51}} \\ &= 0.864\ mA \\ V_C &= 15 - 0.864 \times 10 \\ &= 6.36\ V \end{aligned}$$

and for $\beta = 150$:

$$\begin{aligned} I_C &= \frac{150}{1+150} \times \frac{15 - 0.7}{13 + \frac{164}{151}} \\ &= q.008\ mA \\ V_C &= 15 - 1.008 \times 10 \\ &= 4.92\ V \end{aligned}$$

4.2 Problem 4.49

We wish to design the amplifier circuit of Figure (4.2) under the constraint that V_{CC} is fixed. Let the input signal $v_{be} = \hat{V}_{be} \sin \omega t$ where $\hat{V}_{be}$ is the maximum value for acceptable linearity. Show for the design that results in the largest signal at the collector without the BJT leaving the active region, that

$$R_C I_C = \frac{V_{CC} - V_{BE} - \hat{V}_{be}}{1 + \frac{\hat{V}_{be}}{V_T}}$$

and find an expression for the voltage gain obtained. For $V_{CC} = 10$ V, $V_{BE} = 0.7$ V, and $\hat{V}_{be} = 5$ mV, find the dc voltage at the collector, the amplitude of the output voltage signal, and the voltage gain.

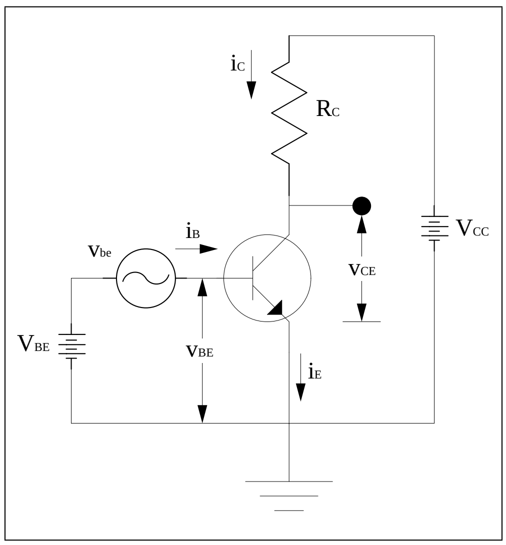

Figure 4.2:

Solution

The total collector current (ac and dc) i_C is given by:

$$\begin{aligned} i_C &= I_C + g_m v_{be} \\ &= I_C + g_m \hat{V}_{be} \sin \omega t \end{aligned}$$

The total collector voltage v_C is similarly given by:

$$v_C = V_{CC} - I_C R_C - g_m \hat{V}_{be} \sin \omega t$$

To maintain the BJT in the active region $v_c \geq v_{be}$ then:

$$V_{CC} - I_C R_C - g_m \hat{V}_{be} \geq V_{BE} + \hat{V}_{be}$$

To maximize v_C we should use the equal sign in the last equation, i.e.

$$V_{CC} - I_C R_C - g_m R_C \hat{V}_{be} = V_{BE} + \hat{V}_{be}$$

Using the expression for g_m:

$$g_m = \frac{I_C}{V_T}$$

the last equation then becomes:

$$\begin{aligned}
V_{CC} - I_C R_C \frac{\hat{V}_{be}}{V_T} &= V_{BE} + \hat{V}_{be} \\
I_C R_C \left(1 + \frac{\hat{V}_{be}}{V_T}\right) &= V_{CC} - V_{BE} - \hat{V}_{be} \\
I_C R_C &= \frac{V_{CC} - V_{BE} - \hat{V}_{be}}{1 + \frac{\hat{V}_{be}}{V_T}}
\end{aligned}$$

The voltage gain $A_v = -g_m R_C$, using the last equation we get:

$$\begin{aligned}
A_v &= g_m R_C \\
&= -\frac{I_C}{V_T} R_C \\
&= -\frac{V_{CC} - V_{BE} - \hat{V}_{be}}{V_T + \hat{V}_{be}} \qquad (4.6)
\end{aligned}$$

Substituting with the given numerical values we get:

$$\begin{aligned}
I_C R_C &= \frac{V_{CC} - V_{BE} - \hat{V}_{be}}{1 + \frac{\hat{V}_{be}}{V_T}} \\
&= \frac{10 - 0.7 - 0.005}{1 + \frac{5}{25}} \\
&= 7.75\ V \\
V_C &= V_{CC} - I_C R_C \\
&= 10 - 7.75 \\
&= 2.25\ V
\end{aligned}$$

The ac output voltage v_c is given by:

$$\begin{aligned} v_c &= V_C - v_{be} \\ &= V_C - (V_{BE} + \hat{V}_{be} \sin \omega t) \end{aligned}$$

The amplitude of v_c will determined by the amplitude of v_{be}, i.e.

$$\begin{aligned} \hat{V}_c &= V_C - (V_{BE} + \hat{V}_{be}) \\ &= 2.25 - (0.7 + 0.005) \\ &= 1.55\ V \end{aligned}$$

The voltage gain can be calculated from $-\hat{V}_c/\hat{V}_{be}$ and from Equation (4.6):

$$\begin{aligned} A_v &= -\frac{\hat{V}_c}{\hat{V}_{be}} \\ &= -\frac{1.55}{0.005} \\ &= -310 \\ &= -\frac{V_{CC} - V_{BE} - \hat{V}_{be}}{V_T + \hat{V}_{be}} \\ &= -\frac{10 - 0.7 - 0.005}{0.025 + 0.005} \\ &= -\frac{9.295}{0.03} \\ &= -309.8 \\ &= -310 \end{aligned}$$

4.3 Problem 4.61

Using the BJT equivalent circuit model of Figure (4.3) sketch the equivalent circuit of a transistor amplifier for which a resistance R_e is connected between the emitter and ground, the collector is grounded and an input signal source v_b is connected between the base and ground. (It is assumed that the transistor is properly biased to operate in the active region.) Show that:

(a) the voltage gain between the base and emitter, that is v_e/v_b, is given by:

$$\frac{v_e}{v_b} = \frac{R_e}{R_e + r_e}$$

(b) the input resistance,

$$R_{in} \equiv \frac{v_b}{i_b} = (\beta + 1)(R_e + r_e)$$

Find the numerical value for (v_e/v_b) and R_{in} for the case $R_e = 1\ k\Omega$, $\beta = 100$ and the emitter bias current $I_E = 1$ mA.

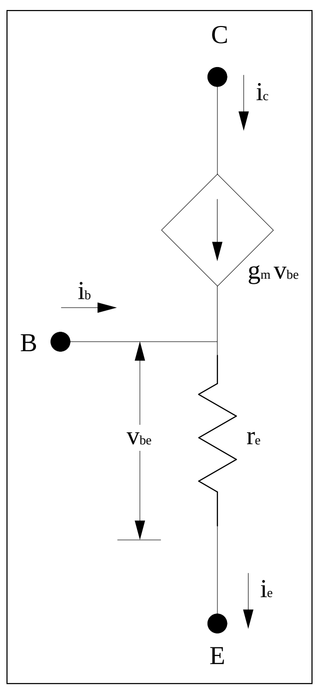

Figure 4.3:

Solution

The required equivalent circuit is shown in Figure (4.4).

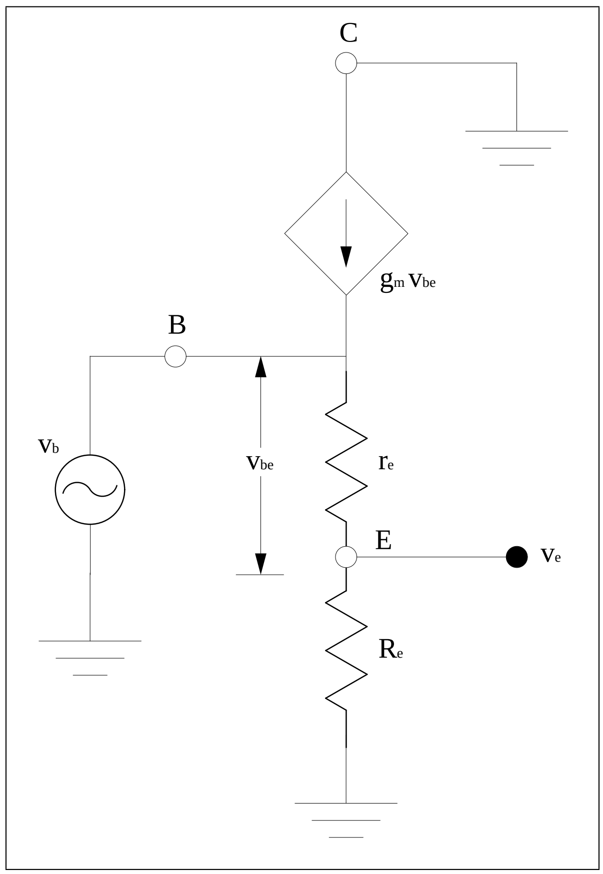

Figure 4.4:

(a) v_b, r_e, and R_e form a voltage divider, where v_e is the voltage across R_e that is given by;

$$v_e = \frac{R_e}{R_e + r_e} v_b$$

$$\frac{v_e}{v_b} = \frac{R_e}{R_e + r_e}$$

(b) the curent equation at the junction at the top of r_e, gives:

$$\begin{aligned}
\frac{v_{be}}{r_e} &= i_b + g_m v_{be} \\
i_b &= \frac{v_{be}}{r_e} - g_m v_{be} \\
&= \frac{v_{be}}{r_e}(1 - g_m r_e) \\
&= \frac{v_{be}}{r_e}\left(1 - g_m \frac{\alpha}{g_m}\right) \\
&= \frac{v_{be}}{r_e}(1 - \alpha) \\
&= \frac{v_{be}}{r_e}\left(1 - \frac{\beta}{1+\beta}\right) \\
&= \frac{v_{be}}{r_e} \times \frac{1}{1+\beta}
\end{aligned}$$

from the volatge divider we get;

$$v_{be} = \frac{r_e}{R_e + r_e} v_b$$

Using this last equation, the base current i_b becomes:

$$\begin{aligned}
i_b &= \frac{1}{r_e(1+\beta)} \times \frac{v_b r_e}{R_e + r_e} \\
&= \frac{1}{1+\beta} \times \frac{v_b}{R_e + r_e} \\
R_i &= \frac{v_b}{i_b} \\
&= (1+\beta)(R_e + r_e)
\end{aligned}$$

Substituting with the given numerical values we get:

$$\begin{aligned}
r_e &= \frac{V_T}{I_E} \\
&= \frac{0.025}{0.001} \\
&= 25\ \Omega \\
\frac{v_e}{v_b} &= \frac{R_e}{R_e + r_e} \\
&= \frac{1000}{1000 + 25} \\
&= 0.976
\end{aligned}$$

$$
\begin{aligned}
R_{in} &= (1+\beta)(R_e + r_e) \\
&= 101 \times 1025 \\
&= 103.5\ k\Omega
\end{aligned}
$$

4.4 Problem 4.83

The amplifier of Figure (4.5) consists of two identical common emitter amplifiers connected in cascade. Observe that the input resistance of the second stage, R_{in2}, constitutes the load resistance of the first stage.

(a) for $V_{CC} = 15$ V, $R_1 = 100\ k\Omega$, $R_2 = 47\ k\Omega$, $R_E = 3.9\ k\Omega$, and $\beta = 100$, determine the dc collector current and collector voltage of each transistor.

(b) Draw the small-signal equivalent circuit of the entire amplifier and give the values of all its components. Neglect $r_{\circ 1}$ and $r_{\circ 2}$.

(c) Find R_{in1} and v_{b1}/v_s for $R_s = 5\ k\Omega$.

(d) Find R_{in2} and v_{b2}/v_{b1}.

(e) For $R_L = 2\ k\Omega$, find v_o/v_{b2}.

(f) Find the overall voltage gain v_o/v_s.

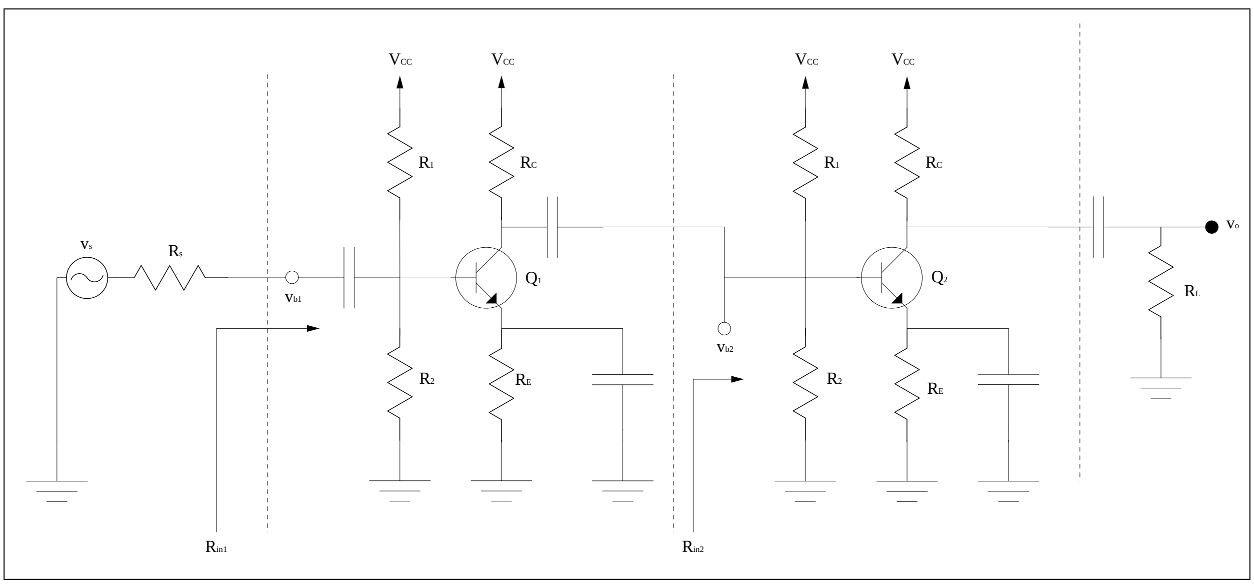

Figure 4.5: All capacitors are blocking capacitors of very large capacitance.

Solution

(a) Since the two stages are identical we then have for each transistor:

$$
\begin{aligned}
V_{BB} &= V_{CC} \times \frac{R_2}{R_1 + R_2} \\
&= 15 \times \frac{47}{100 + 47} \\
&= 4.8\ V \\
R_B &= R_1 // R_2 \\
&= 100 // 47 \\
&= 32\ k\Omega \\
I_E &= \frac{V_{BB} - V_{BE}}{R_E + \frac{R_B}{1+\beta}} \\
&= \frac{4.8 - 0.7}{3.9 + \frac{32}{101}} \\
&= 0.97\ mA \\
I_C &= \alpha I_E \\
&= \frac{\beta}{1+\beta} \times I_E \\
&= \frac{100}{101} \times 0.97 \\
&= 0.96\ mA
\end{aligned}
$$

(b) The small signal equivalent circuit is shown in Figure (4.6). Once again, since the two

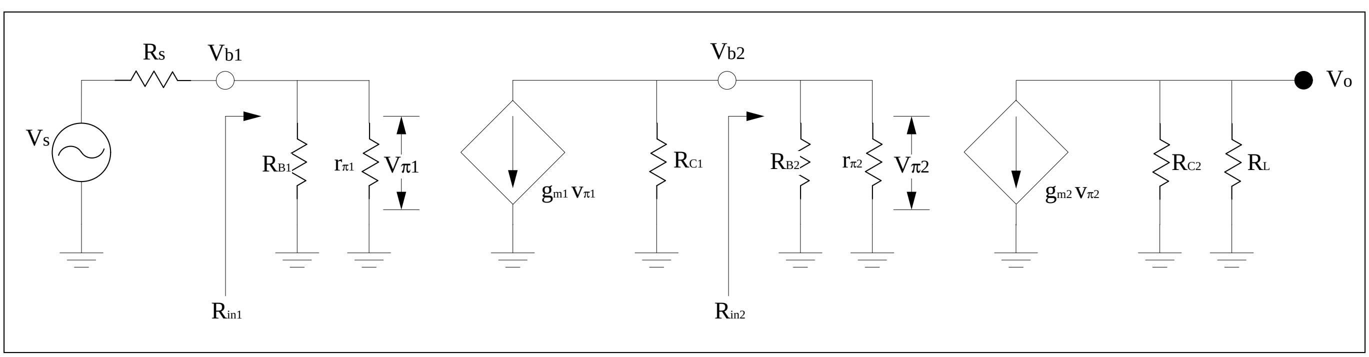

Figure 4.6:

stages are identical, we get:

$$\begin{aligned} R_{B1} &= R_{B2} = R_B \\ &= 32\ k\Omega \\ g_{m1} &= g_{m2} \\ &= \frac{I_C}{V_T} \\ &= \frac{0.96}{0.025} \\ &= 38.4\ mV/A \\ r_{\pi 1} &= r_{\pi 2} \\ &= \frac{\beta}{g_m} \\ &= \frac{100}{38.4} \\ &= 2.6\ k\Omega \\ R_{C1} &= R_{C2} \\ &= 6.8\ k\Omega \end{aligned}$$

(c)

$$\begin{aligned} R_{in1} &= R_{B1}//r_{\pi 1} \\ &= 32//2.6 \\ &= 2.4\ k\Omega \end{aligned}$$

Using the voltage divider formed by v_s, R_s, and R_{in1}, we get:

$$\begin{aligned} v_{b1} &= \frac{R_{in1}}{R_s + R_{in1}} \times v_s \\ \frac{v_{b1}}{v_s} &= \frac{2.4}{5 + 2.4} \\ &= 0.32 \end{aligned}$$

(d)

$$\begin{aligned} R_{in2} &= R_{B2}//r_{\pi 2} \\ &= 32//2.6 \\ &= 2.4\ k\Omega \end{aligned}$$

v_{b2} is the voltage produced by the current $g_{m1}v_{\pi 1}$ flowing through the parallel equivalent of R_{C1}, R_{B2}, and $r_{\pi 2}$, notice that $v_{\pi 1} = v_{b1}$, so:

$$\begin{aligned}
v_{b2} &= -g_{m1}v_{\pi 1} \times R_{C1}//R_{B2}//r_{\pi 2} \\
&= -g_{m1}v_{b1} \times R_{C1}//R_{in1} \\
&= -34.4 \times v_{b1} \times (6.8//2.4) \\
\frac{v_{b2}}{v_{b1}} &= -68.1
\end{aligned}$$

(e) Similarly, v_o is given by:

$$\begin{aligned}
v_o &= -g_{m2}v_{\pi 2} \times (R_{C2}//R_L) \\
&= -g_{m2}v_{b2} \times (R_{C2}//R_L) \\
\frac{v_o}{v_{b1}} &= -34.4 \times (6.8//2.0) \\
&= -59.3
\end{aligned}$$

(f) The overall gain v_o/v_s is given by:

$$\begin{aligned}
\frac{v_o}{v_s} &= \frac{v_{b1}}{v_s} \times \frac{v_{b2}}{v_{b1}} \times \frac{v_o}{v_{b2}} \\
&= 0.32 \times -68.1 \times -59.3 \\
&= 1292
\end{aligned}$$

4.5 Problem 4.92

In the emitter follower in Figure (4.7), the signal source is directly coupled to the transistor base. If the dc component of v_s is zero, find the dc emitter current. Asume $\beta = 120$. Neglecting $r_\circ$, find R_i, the voltage gain $v_\circ/v_s$, the current gain $i_\circ/i_s$ and the output resistance $R_\circ$.

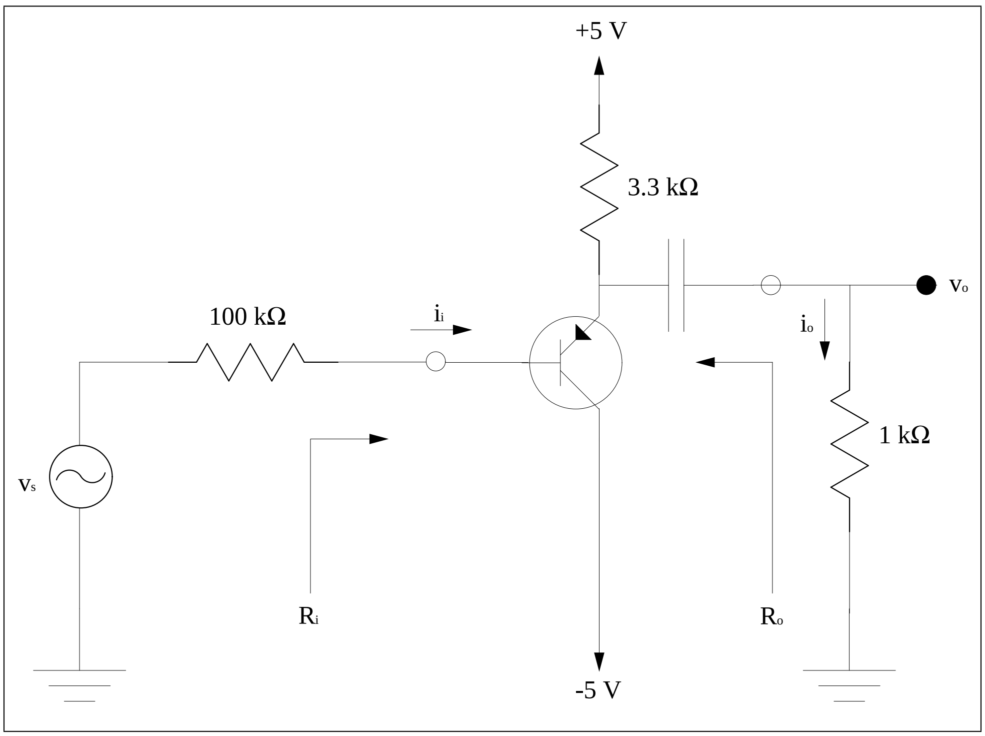

Figure 4.7: The capacitor is a blocking capacitor of very large capacitance.

Solution

The T-model equivalent of the given circuit is shown in Figure (4.8)
Given that $\alpha \approx 1$, the emitter current I_E is given by:

$$\begin{aligned} I_E &= \frac{V_{CC} - V_{BE}}{R_C + \frac{R_B}{1+\beta}} \\ &= \frac{5.0 - 0.7}{3.3 + \frac{100}{121}} \\ &= 1.042\ mA \end{aligned}$$

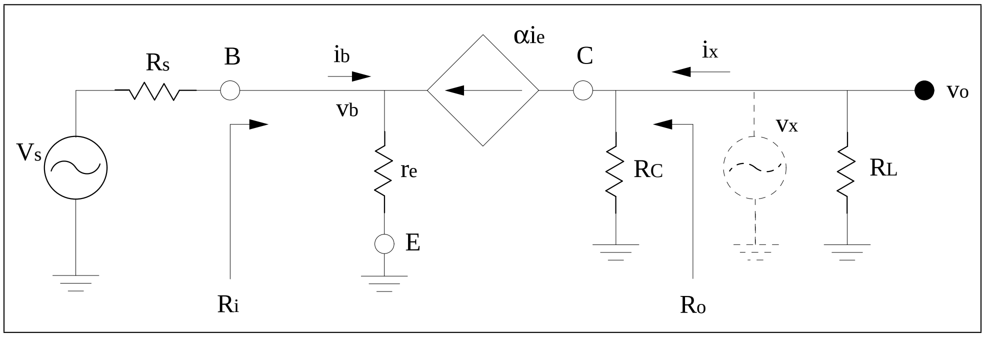

Figure 4.8:

We can calculate r_e and R_i from:

$$\begin{aligned} r_e &= \frac{V_T}{I_E} \\ &= \frac{25}{1.042} \\ &= 24\ \Omega \end{aligned}$$

The input resistance R_i is the resistance that the source will see looking into the base. It is clear from Figure (4.8) that R_i is composed of r_e, R_C, and R_L. The last two resistors are connected in parallel and obviously the $R_{CL} = R_C//R_L$ is in series with r_e since they both carry the same current. This situation is similar to that where a resistor R_e is connected to the emitter and is in series with r_e, in this case $R_i = (1 + \beta)(r_e + R_e)$. In the case at hand R_i is then given by:

$$\begin{aligned} R_i &= (1+\beta)(r_e + R_{CL}) \\ &= (1+\beta)(r_e + R_C//R_L) \\ &= 121 \times \left(24 + \frac{3.3 \times 1}{3.3 + 1}\right) \\ &= 121 \times (24 + 767) \\ &= 95.8\ k\Omega \end{aligned}$$

v_b, r_e, and R_{CL} form a voltage divider. The output voltage v_o is the voltage across R_{CL} we then have:

$$\frac{v_o}{v_b} = \frac{R_{CL}}{r_e + R_{CL}}$$

while v_s, R_s, and R_i form another voltage divider where v_b is the voltage across R_i, we then have:

$$\frac{v_b}{v_s} = \frac{R_i}{R_s + R_i}$$

Using the last two equations, the overall voltage gain v_o/v_s is:

$$\begin{aligned}\frac{v_o}{v_s} &= \frac{v_b}{v_s} \times \frac{v_o}{v_b} \\ &= \frac{R_i}{R_s + R_i} \times \frac{R_{CL}}{r_e + R_{CL}} \\ &= \frac{95.8}{100 + 95.8} \times \frac{0.767}{0.024 + 0.767} \\ &= 0.474\end{aligned}$$

The input current i_i is the current produced by the input voltage v_s in the series combination of R_s and R_i, while the output current i_o is produced by the output voltage through the load resistor R_L, so the overall current gain i_o/i_i is given by:

$$\begin{aligned}\frac{i_o}{i_i} &= \frac{v_o}{R_L} / \frac{v_s}{R_s + R_i} \\ &= \frac{v_o}{v_s} \times \frac{R_s + R_i}{R_L} \\ &= 0.474 \times \frac{100 + 95.8}{1} \\ &= 92.8\end{aligned}$$

To find the output resistance R_o we set v_s to zero and insert a virtual voltage source v_x at the point where the load device looks back at the circuit. Let us assume that v_x produces a virtual current i_x, as shown by the dashed part of the circuit in Figure (4.8). Taking v_x across the input part of the circuit ($v_s = 0$), we get:

$$\begin{aligned}v_x &= i_e r_e + i_b R_s \\ &= i_e r_e + (1 - \alpha) i_e R_s \\ &= i_e r_e + \frac{R_s}{1 + \beta} \\ &= i_e \left[r_e + \frac{R_s}{1 + \beta} \right]\end{aligned}$$

The virtual current i_x is given by:

$$\begin{aligned}
i_x &= \frac{v_x}{R_C} + i_e \\
&= \frac{v_x}{R_C} + \frac{v_x}{r_e + \frac{R_s}{1+\beta}} \\
\frac{i_x}{v_x} &= \frac{1}{R_o} \\
&= \frac{1}{R_C} + \frac{1}{r_e + \frac{Rs}{1+\beta}} \\
R_o &= R_C // \left[r_e + \frac{R_s}{1+\beta} \right] \\
&= 3.3 // \left[0.024 + \frac{100}{121} \right] \\
&= 3.3//0.85\ k\Omega \\
&= \frac{3.3 \times 0.85}{3.3 + 0.85} \\
&= 0.676\ k\Omega
\end{aligned}$$

4.6 Problem 4.96

For the follower circuit in Figure (4.9) let transistor Q_1 have $\beta = 20$ and transistor Q_2 have $\beta = 200$, and neglect the effect of $r_\circ$. Use $V_{BE} = 0.7$ V.

(a) Find the dc emitter current of Q_1 and Q_2. Also find the dc voltages V_{B1} and V_{B2}.

(b) If a load resistance $R_L = 1\ k\Omega$, is connected to the output terminal, find the voltage gain from the base to the emitter of Q_2, $v_\circ/v_{b2}$, and find the input resistance R_{ib2} looking into base of Q_2. (*Hint:* Consider Q_2 as an emitter follower fed by a voltage v_{b2} at its base.)

(c) Replacing Q_2 with its input resistance R_{ib2} found in (b), analyze the circuit of emitter follower Q_1 to determine its input resistance R_i, and the gain from its base to its emitter, v_{e1}/v_{b1}.

(d) If the circuit is fed with a source having a 100-$k\Omega$ resistance, find the transmission to the base of Q_1, v_{b1}/v_s.

(e) Find the overall voltage gain $v_\circ/v_s$.

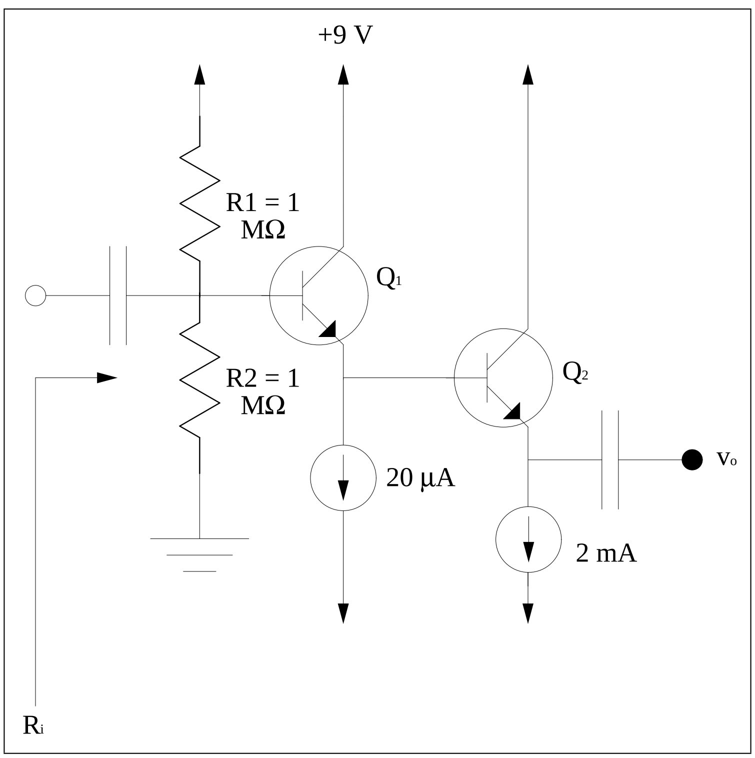

Figure 4.9: The capacitors are a blocking capacitors of very large capacitance.

Solution

(a) In the base circuit of Q_1, one can replace V_{CC}, $R_1 = 1M\Omega$, $R_2 = 1M\Omega$ by their the Thevenin's equivalent of R_{BB} and V_{BB}, such that:

$$\begin{aligned}
R_{BB} &= \frac{R_1 R_2}{R_1 + R_2} \\
&= \frac{1 \times 1}{1 + 1} \\
&= 0.5\ M\Omega \\
V_{BB} &= V_{CC} \times \frac{R_1}{R_1 + R_2} \\
&= 9.0 \times 0.5 \\
&= 4.5\ V
\end{aligned}$$

The emitter currents of Q_1 and Q_2 are given by:

$$\begin{aligned}
I_{E1} &= 2\ mA \\
I_{E2} &= 20\ \mu A + I_{B2} \\
&= 20\ \mu A + \frac{I_{E2}}{1 + \beta_2} \\
&= 20\ \mu A + \frac{2000(\mu A)}{201} \\
&= 30\ \mu A
\end{aligned}$$

The base voltages of Q_1 and Q_2, are:

$$\begin{aligned}
V_{B1} &= V_{BB} - I_{B1} R_{BB} \\
&= V_{BB} - \frac{I_{E1} 1 + \beta_1}{\times} R_{BB} \\
&= 4.5 - \frac{30(\mu A)}{21} \times 0.5(M\Omega) \\
&= 4.5 - 1.43(\mu A) \times 0.5(M\Omega) \\
&= 3.79\ V \\
V_{B2} &= V_{B1} - V_{BE} \\
&= 3.79 - 0.7 \\
&= 3.09\ V
\end{aligned}$$

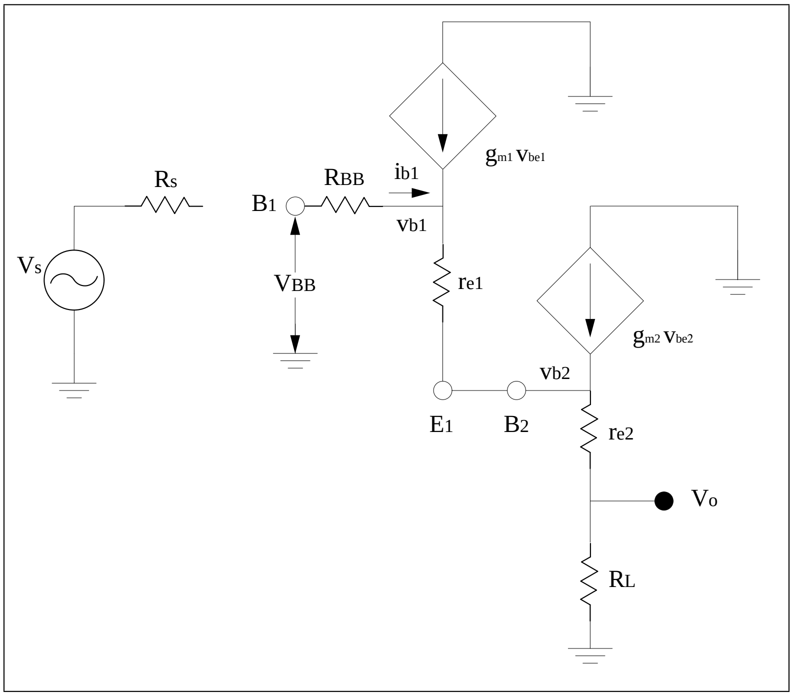

Figure 4.10:

(b) the T-model equivalent of the whole circuit is shown in Figure (4.10). It is clear from the figure that:

$$
\begin{aligned}
v_o &= \frac{R_L}{R_L + r_{e2}} \times v_{b2} \\
r_{e2} &= \frac{V_T}{I_{E2}} \\
&= \frac{25}{2} \\
&= 12.5\ \Omega \\
\frac{v_o}{v_{b2}} &= \frac{R_L}{R_L + r_{e2}} \\
&= \frac{1000}{1000 + 12.5} \\
&= 0.988 \\
R_{ib2} &= (1 + \beta_2)(r_{e2} + R_L) \\
&= 201 \times (1000 + 12.5) \\
&= 203.5\ k\Omega
\end{aligned}
$$

(c) Replacing the second transistor Q_2 by its input resistance in Figure (4.10)we get:

$$\begin{aligned}
r_{e1} &= \frac{V_T}{I_{E1}} \\
&= \frac{25000(\mu V)}{30(\mu A)} \\
&= 833\ \Omega \\
&= 0.833\ k\Omega \\
v_{e1} &= \frac{R_{ib2}}{R_{ib2} + r_{e1}} \times v_{b1} \\
\frac{v_{e1}}{v_{b1}} &= \frac{R_{ib2}}{R_{ib2} + r_{e1}} \\
&= \frac{203.5}{203.5 + 0.833} \\
&= 0.996 \\
R_i &= R_{BB} // (1 + \beta_1)(r_{e1} + R_{ib2}) \\
&= 500 // [21 \times (.833 + 203.5)]\ k\Omega \\
&= 0.5 // 4.29\ M\Omega \\
&= 0.448\ M\Omega \\
&= 448\ k\Omega
\end{aligned}$$

(d) In Figure (4.10) let us connect v_s with it internal resistance $R_s = 100\ k\Omega$, and replaceing Q_1 by its internal resistance R_i we get:

$$\begin{aligned}
\frac{v_{b1}}{v_s} &= \frac{R_i}{R_i + R_s} \\
&= \frac{448}{448 + 100} \\
&= 0.818
\end{aligned}$$

(e) finally the overall gain is (note that $v_{e1} = v_{b2}$):

$$\begin{aligned}
\frac{v_o}{v_s} &= \frac{v_{b1}}{v_s} \times \frac{v_{e1}}{v_{b1}} \times \frac{v_o}{v_{b2}} \\
&= 0.818 \times 0.996 \times 0.988 \\
&= 0.805
\end{aligned}$$